

Tree-level stability without spacetime fermions: novel examples in string theory

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ABSTRACT: Is perturbative stability intimately tied with the existence of spacetime fermions in string theory in more than two dimensions? Type 0'B string theory in ten-dimensional flat space is a rare example of a non-tachyonic, non-supersymmetric string theory with a purely bosonic closed string spectrum. However, all known type 0' constructions exhibit massless NSNS tadpoles signaling the fact that we are not expanding around a true vacuum of the theory. In this note, we are searching for perturbatively stable examples of type 0' string theory without massless tadpoles in backgrounds with a spatially varying dilaton. We present two examples with this property in non-critical string theories that exhibit four- and six-dimensional Poincaré invariance. We discuss the D-branes that can be embedded in this context and the type of gauge theories that can be constructed in this manner. We also comment on the embedding of these non-critical models in critical string theories and their holographic (Little String Theory) interpretation and propose a general conjecture for the role of asymptotic supersymmetry in perturbative string theory.

KEYWORDS: Superstring Vacua, Conformal Field Models in String Theory, D-branes.

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1. Introduction and summary

Supersymmetry has been a major driving force in theoretical physics for the last two or three decades. It is not only a conceptually appealing construction and a powerful calculational tool, but is most likely also an important ingredient of the real world. At the same time, there is an obvious interest in situations where supersymmetry has been broken mildly or is altogether absent. In this note, we will be interested in string theory examples where the spectrum of closed strings is purely bosonic, so supersymmetry in the bulk is broken to the highest degree. Weakly coupled string theories with this feature are expected to be relevant for the holographic description of certain non-supersymmetric large N gauge theories, for instance large N QCD.

A well known example of a string theory with a purely bosonic closed string spectrum is type 0 string theory, but in this case the spectrum contains a closed string tachyon signaling the perturbative instability of the vacuum. The type 0 closed string tachyon can be projected out of the physical spectrum in a non-oriented version of this theory, which is known as the type 0'B theory [1–3]. However, typically in this case there are massless NSNS tadpoles at tree-level which drive the theory towards the true vacuum via the Fischler-Susskind mechanism [4–6]. In this new vacuum, which may or may not be perturbative, the shifted background is expected to have a non-trivial metric and dilaton. In fact, in cases where the backreaction of the NSNS tadpoles has been analyzed [7, 8] (see also [9] for a recent discussion) it was found that the system is naturally driven at strong coupling. Furthermore, at higher genus, one obtains additional tadpoles for the dilaton as an immediate consequence of the absence of supersymmetry.

As a simple attempt to get rid of the massless tadpoles, we consider in section 2 type 0B string theory compactified on $\mathbb{R}^{8,1} \times S^1$ with a modified worldsheet parity projection that acts non-trivially on the S^1 . This example involves a space-filling $O'9$ orientifold, constructed in such a way that it sources only massive RR fields. Although the ground state of the NSNS sector (with zero momentum and winding along the S^1) is projected out, a subset of more massive states with non-vanishing momentum and/or winding remains. It turns out that for any value of the compactification radius there are tachyonic modes that cannot be projected out. This paradigm demonstrates how non-trivial it is, in general, to obtain an example in type 0' string theory that is at the same time non-tachyonic and does not exhibit massless tadpoles, *i.e.* that exhibits full tree-level stability.

The generic presence of dilaton tadpoles in the above examples motivates the study of type 0' string theories in backgrounds with a non-trivial dilaton profile. In section 3, we present the main result of this note, namely type 0' string theories in non-critical dimensions, with an asymptotic linear dilaton, that are perturbatively stable. Similar to the ten-dimensional model discussed in section 2, these are also situations with space-filling orientifolds sourcing only massive RR fields. However, the spectrum in this case is non-tachyonic. The orientifold projection removes the closed string tachyons from the continuous representations (closed strings that propagate along the linear dilaton direction). The models we consider have no localized tachyons, so the whole closed string spectrum is tachyon-free. The origin of this crucial difference with the standard examples of type 0' lies in the presence of the background charge of the linear dilaton, that gives a universal positive mass shift to the spectrum, and the non-trivial diagonal GSO projection in non-critical superstrings (for the six dimensional example).

The vacua presented in section 3 are, to our knowledge, the first *bona fide* examples of weakly coupled string theories with a purely bosonic closed string spectrum and full tree-level stability (*i.e.* are tachyon- and tadpole-free). Are these examples special isolated cases, or part of a more general construction with the same features? The possibility of more general type 0' vacua with a linear dilaton is discussed in the concluding section. To address this issue we discuss the embedding of these vacua in ten-dimensional string theory and their relation with fivebrane configurations and Little String Theory. We find that our non-critical type 0' vacua arise as an equivalent description of the near-horizon limit of NS5-branes configurations in ten-dimensional type 0 string theories in the presence of orientifold planes. For instance, the six-dimensional model corresponds to the double scaling limit of a pair of parallel fivebranes in type 0A with an $O'6$ -plane at an equal distance between them.

This correspondence between non-critical type 0'B and fivebranes/orientifold configurations leads to a number of interesting observations. First, in the near-horizon geometry of more than two parallel fivebranes, the $O'6$ -plane does not succeed in fully projecting out the tachyon. Perturbative stability in these models is only possible deep in the stringy regime, for a pair of fivebranes. Secondly, the non-critical models at hand correspond to fivebranes/orientifold configurations immersed in a type 0A bulk that contains a closed string tachyon. In the near-horizon decoupling limit of two fivebranes only an s -wave mode of the tachyon remains and this is projected out by the orientifold.

In the last part of the concluding section, we elaborate on the general role of spacetime fermions and supersymmetry in perturbatively stable string theory and propose a new conjecture that links perturbative stability and asymptotic supersymmetry generalizing previous arguments in [10–12]–[13] in accordance with the specific examples presented in this note.

Another motivation behind this work is holography for non-supersymmetric gauge theories. For instance, by placing N D3-branes on top of an $O'3$ -plane in ten-dimensional flat spacetime, one obtains on the D-branes a non-supersymmetric gauge theory with $U(N)$ gauge group. The AdS/CFT correspondence for this theory, which is similar in spirit with the type 0 AdS/CFT proposal of [14], was discussed in [15]. Further examples of this sort, in the presence or absence of bulk tachyons, can be found in [16, 15, 17] and references therein. D-branes in the type IIA variant of the non-critical string theories considered in this note can be used to engineer interesting four dimensional gauge theories like $\mathcal{N} = 1$ SQCD [18]. Investigating the D-branes of the four-dimensional non-critical type $0'$ string theory of section 3 we find that the corresponding D-branes are unstable and give rise to four-dimensional non-supersymmetric gauge theories in the presence of an open string tachyon. Nevertheless, it is possible in this context to engineer odd dimensional (flavored) non-supersymmetric gauge theories by using D-branes of a different dimensionality. Non-supersymmetric four-dimensional gauge theories can be realized on D-branes without open string tachyons in the six-dimensional type $0'$ model. We comment on the various possibilities in section 3.

2. Critical type $0'$ B strings

2.1 Overview

Tachyon-free type 0 models in ten-dimensional critical string theory were first discussed in [1–3]. In the original example, the worldsheet theory of type 0B strings is modded out by the modified worldsheet parity $\Omega' = (-)^{\bar{F}}\Omega$, where \bar{F} denotes the right-moving worldsheet fermion number. This parity acts in the closed string sector as¹

$$\Omega' \alpha_n \Omega' = \bar{\alpha}_n, \quad \Omega' \psi_r \Omega' = \bar{\psi}_r, \quad \Omega' \bar{\psi}_r \Omega' = \psi_r, \quad \Omega' |0\rangle_{\text{NSNS}} = -|0\rangle_{\text{NSNS}} \quad (2.1)$$

and projects out the tachyonic ground state in the NSNS sector, which is part of the type 0B spectrum. In spacetime, this action introduces a space-filling $O'9$ -plane, which is a source for the non-dynamical RR ten-form. Consistency requires the cancellation of this tadpole. This can be achieved with the addition of thirty two D9- and D9'-branes (for a detailed description of D-branes in type 0B string theory see [19]). However, the addition of D-branes leads to an uncancelled dilaton tadpole from the NSNS sector. This tadpole does not render the theory inconsistent, but shows that we are not expanding around a true vacuum of the theory. Since it is hard to determine the true vacuum (*e.g.* by implementing the Fischler-Susskind mechanism) and because the true vacuum may be naturally at strong coupling, it is unclear to what extent we can read off the true properties

¹By definition Ω acts on fermion bilinears as $\psi_r \bar{\psi}_r \rightarrow \psi_r \bar{\psi}_r$ and Ω' as $\psi_r \bar{\psi}_r \rightarrow -\psi_r \bar{\psi}_r$.

of the theory from the above construction (see however [20] for a more recent discussion of tadpoles and vacuum redefinitions in string theory).

Despite this annoying feature, the above construction leads to a very interesting non-supersymmetric model. In the bulk, this is a purely bosonic theory of unoriented closed strings with a massless spectrum, which is essentially a bosonic truncation of the type IIB spectrum. The only difference is the absence of the NSNS two-form field, which is projected out by Ω' . On the D9-, D9'-branes one obtains a bosonic spectrum from open strings stretching between the same type of branes and a fermionic spectrum from open strings stretching between a D9 and a D9'. At the massless level, the former give a $U(32)$ gauge theory and the latter a Majorana-Weyl fermion in the $\mathbf{496} \oplus \overline{\mathbf{496}}$ of $U(32)$.

An exhaustive analysis of tachyon-free orbifold compactifications has been performed in [21–23]–[24] (for a review see [25]). Typically in these examples, one finds twisted sector closed string tachyons that cannot be projected out by any generalized Ω projection. There are certain cases, however, where twisted sector tachyons are absent and non-tachyonic vacua can be constructed [25].

The presence of massless NSNS and RR tadpoles is a generic feature in these examples. Canceling the RR tadpoles requires the addition of D-branes and one is usually left with a theory of open and closed strings in the presence of an uncanceled dilaton tadpole. Are massless tadpoles at tree level an essential property of all type 0' vacua? Is it always necessary to add a certain number of D-branes? The answer to both of these questions is no. In the next section, we present two examples of non-critical type 0' closed strings in more than two dimensions, where no tachyons and no massless tadpoles exist and no D-branes need to be added for consistency. We are not aware of a similar example on backgrounds with a constant dilaton. To illustrate the rarity of such theories, we present in the next subsection a 0' orientifold of type 0B theory on $\mathbb{R}^{8,1} \times S^1$, which does not exhibit any massless tadpoles, but fails to fully project out the zero modes of the closed string tachyon.

2.2 A model without massless tadpoles but with tachyons

Let us consider type 0B string theory on $\mathbb{R}^{8,1} \times S^1$.² This theory has the torus partition function

$$\mathcal{T} = \frac{\mathcal{V}_{10}}{2} \int_{\mathcal{F}} \frac{d\tau d\bar{\tau}}{4\tau_2} \frac{1}{(8\pi^2\tau_2)^{9/2}} \frac{1}{\eta^8 \bar{\eta}^8} \sum_{n,w=-\infty}^{\infty} q^{\frac{1}{2}(\frac{n}{R} + \frac{wR}{2})^2} \bar{q}^{\frac{1}{2}(\frac{n}{R} - \frac{wR}{2})^2} \sum_{a,b \in \mathbb{Z}_2} \frac{\vartheta^A[\frac{a}{b}]\bar{\vartheta}^A[\frac{a}{b}]}{\eta^4 \bar{\eta}^4} \quad (2.2)$$

where R denotes the S^1 radius, \mathcal{V}_{10} the overall volume diverging factor, τ the modular parameter of the torus, $q = e^{2\pi i\tau}$ and \mathcal{F} is the $PSL(2, \mathbb{Z})$ fundamental domain. The modular functions $\vartheta[\frac{a}{b}](\tau)$, $\eta(\tau)$ are the standard ϑ - and η -functions.

As an attempt to obtain a model similar to type 0'B but with a massive RR tadpole, we want to mod out this theory with the worldsheet parity

$$\mathcal{P} = s(-)^{\bar{F}} \Omega, \quad (2.3)$$

²The example in this subsection was suggested to us by Carlo Angelantonj.

where s is a shift symmetry acting on X , the U(1) boson of radius R , that parametrizes the S^1 part of the background:

$$s : X \rightarrow X + \pi R . \tag{2.4}$$

In the NSNS sector, \mathcal{P} acts on the states $|n, w\rangle_{\text{NSNS}}$ (with momentum n and winding w around the S^1) as

$$\mathcal{P} : |n, w\rangle \rightarrow (-)^{n+1} |n, -w\rangle . \tag{2.5}$$

In particular, \mathcal{P} projects out all the NSNS ground states with even momentum and zero winding. The lightest allowed modes in the NSNS sector are (i) the antisymmetric combination of the modes with zero momentum and winding number $w = \pm 1$ and (ii) states with momentum $n = \pm 1$ and zero winding. The winding (respectively momentum) modes have mass squared (in our conventions $\alpha' = 2$)

$$m_{w=\pm 1}^2 = -1 + \frac{R^2}{4}, \quad m_{n=\pm 1}^2 = -1 + \frac{1}{R^2} . \tag{2.6}$$

It is clear from these expressions that there is no value of the radius R where both $m_{w=\pm 1}^2$ and $m_{n=\pm 1}^2$ are simultaneously positive. Hence, the bulk tachyon cannot be projected out fully in this example.

In the Klein bottle amplitude only states with zero winding contribute. In the direct channel one obtains

$$\mathcal{K} = \frac{\mathcal{V}_{10}}{2} \int_0^\infty \frac{dt}{2t} \frac{1}{(8\pi^2 t)^{9/2}} \frac{1}{\eta^8(2it)} \sum_{n \in \mathbb{Z}} (-)^{n+1} e^{-2\pi t \frac{n^2}{R^2}} \sum_{a \in \mathbb{Z}_2} (-)^a \frac{\vartheta^4 \left[\begin{smallmatrix} a \\ 1 \end{smallmatrix} \right] (2it)}{\eta^4(2it)}, \tag{2.7}$$

where only the $\widetilde{\text{NSNS}}$ and $\widetilde{\text{RR}}$ sectors appear. Using the well-known modular transformation properties of the ϑ - and η -functions and the Poisson resummation formula we can easily deduce the expression of the Klein bottle amplitude in the transverse crosscap channel ($l = \frac{1}{2t}$)

$$\widetilde{\mathcal{K}} = -\frac{\mathcal{V}_{10}}{2} \frac{R}{2(2\pi)^9} \int_0^\infty dl \frac{1}{\eta^8(il)} \sum_{w \in 2\mathbb{Z}+1} e^{-\pi l \frac{w^2 R^2}{4}} \sum_{a \in \mathbb{Z}_2} (-)^a \frac{\vartheta^4 \left[\begin{smallmatrix} 1 \\ a \end{smallmatrix} \right] (il)}{\eta^4(il)}. \tag{2.8}$$

In this channel only odd winding modes in the RR sector contribute. Since all of these modes are massive, we conclude that the O'9-plane presented here has no massless tree level tadpoles. The non-tachyonic, type O'B theories that will be presented below, share many similar features with this ten-dimensional example.

We can gain intuition about the geometry of this O'9 orientifold by looking at a similar example in the conformal field theory of a single U(1) boson X with radius R . In that context, one can formulate four basic parities: the usual worldsheet parity Ω , the parity $s\Omega$, where s is again given by (2.4), and the parities $\mathcal{I}\Omega$ and $\mathcal{I}'\Omega$, which can be obtained by T-duality from the Ω , $s\Omega$ parities of the $2/R$ theory [26]. Both $\mathcal{I}\Omega$ and $\mathcal{I}'\Omega$ include the involution $X \rightarrow -X$ and they have two fixed points, one at $X = 0$ and the other at $X = \pi R$. So, both parities involve a pair of O0-planes. In the $\mathcal{I}\Omega$ case, the orientifold

planes have the same tension, whereas in the $\mathcal{I}'\Omega$ case they have opposite tension. In the case analyzed in this subsection, we are dealing essentially with its T-dual, an $s\Omega$ parity, which, in the S^1 part of the theory, we can think of as a combination of two circle-wrapping $O1$ -planes with opposite RR charge.

3. Non-critical type $O'B$ strings

Given the natural presence of dilaton tadpoles in non-supersymmetric vacua (at tree level and beyond), it is, as explained in the introduction, a good idea to look for stable non-supersymmetric vacua with a non-trivial dilaton background. In this section, which contains the main results of this note, we will concentrate on backgrounds that possess asymptotically a linear dilaton. In general, these backgrounds have a weak coupling asymptotic region where the tree-level analysis is reliable (as the string coupling constant falls off exponentially there) and a strong coupling end where non-trivial physics takes place. In the precise setup that will be analyzed below, the strong coupling region is effectively cut-off by the addition of a non-trivial momentum condensate. The genus expansion is then controlled by an effective string coupling related to the value of the condensate. With appropriate tuning string theory in these vacua becomes perturbative and the higher loop backreaction is accordingly suppressed everywhere and not just in the asymptotic region.

Backgrounds with an asymptotic linear dilaton direction are interesting for a number of reasons. It has been argued on general grounds [27] that string theory on these backgrounds is holographically dual to a non-gravitational theory. A well known class of examples arises in type II string theory, where linear dilaton backgrounds appear in the near-horizon geometry of NS5-brane configurations [28, 29]. The holographic dual in this case is a non-local, non-gravitational theory known as Little String Theory [30].

Another well known example is that of two dimensional non-critical strings. Type 0 strings in two dimensions have been discussed from the worldsheet and matrix model point of view in [31, 32]. In this case string theory is both perturbatively and non-perturbatively stable. Orientifolds in this context have been discussed in [33, 34]. For the $O\mathbb{B}/\Omega$ orientifold, one finds no massless tadpoles from the Klein bottle. For all other parities, there is a tachyon tadpole.³ Interestingly enough, it was pointed out in [33] that the dual matrix model appears to describe string propagation in a shifted background, where the string divergences have been cancelled via the Fischler-Susskind mechanism. From the worldsheet point of view one could attempt to cancel the tachyon tadpoles explicitly with the addition of the appropriate number of D1-branes [34]. The D1-branes cancel the massless one-loop divergence in the leading volume diverging piece of the one-loop amplitude, but leave a finite uncanceled leftover [35]. It was pointed out in [33, 34] that there is no type $O'B$ $\hat{c} = 1$ string in two dimensions, because the $\mathcal{N} = 1$ Liouville potential is odd under $(-)^{\hat{F}}\Omega$. We will see that this obstruction can be evaded in a set of higher dimensional examples which we now proceed to discuss.

³The tachyon in two dimensions, which is the only physical propagating degree of freedom, is a massless scalar field.

In what follows, we will examine the possibility of stable type $0'$ vacua in non-critical string theories in higher dimensions. In that case the NSNS ground state is really a tachyon that needs to be projected out in order to define a perturbatively stable theory. In this note we will focus on a class of non-critical string theories that were first considered in [36] (for a review see [37]). These are string theories on

$$\mathbb{R}^{d-1,1} \times [(\mathcal{N} = 2 \text{ Liouville}) \times \mathcal{M}] / \Gamma, \tag{3.1}$$

where $d \leq 8$ is an even positive integer, Γ is some discrete group and \mathcal{M} is the compact target space of some two-dimensional CFT with $\mathcal{N} = (2, 2)$ worldsheet supersymmetry (e.g. a product of supersymmetric minimal models or Landau Ginzburg models).

The common factor of these backgrounds is $\mathcal{N} = 2$ Liouville theory. On the worldsheet the latter is a theory of two interacting bosons r, φ and two fermions ψ, ψ^\dagger .⁴ The $\mathcal{N} = 2$ Liouville action in superspace notation (and $\alpha' = 2$ conventions) reads

$$\mathcal{S} = \frac{1}{8\pi} \int d^2z d^4\theta \Phi \Phi^\dagger + \frac{\mu}{2\pi} \int d^2z d\theta d\bar{\theta} e^{-\sqrt{\frac{k}{2}}\Phi} + \frac{\mu^\dagger}{2\pi} \int d^2z d\theta^\dagger d\bar{\theta}^\dagger e^{-\sqrt{\frac{k}{2}}\Phi^\dagger}. \tag{3.2}$$

In this expression Φ is the chiral $\mathcal{N} = 2$ superfield

$$\Phi = r + i\varphi + i\sqrt{2}\theta\psi + i\sqrt{2}\bar{\theta}\bar{\psi} + 2\theta\bar{\theta}F + \dots. \tag{3.3}$$

The boson r parametrizes a linear dilaton direction with linear dilaton slope $\mathcal{Q} = \sqrt{\frac{2}{k}}$. The central charge of the $\mathcal{N} = 2$ Liouville theory is $c = 3(1 + \mathcal{Q}^2)$, so the real parameter k that appears in \mathcal{Q} and (3.2) is fixed by the criticality condition $c_{\text{tot}} = 15$, where c_{tot} is the total central charge of the worldsheet theory (3.1). The compact $U(1)$ boson φ has the possible radii $R = n\mathcal{Q}$, $n = 1, 2, \dots$, such that the superpotential has the right periodicity properties.

By construction the background (3.1) exhibits $\mathcal{N} = (2, 2)$ worldsheet supersymmetry. At the special radius $R = \mathcal{Q}$ one can construct spacetime supercharges which are mutually local with all the vertex operators of a chirally GSO-projected theory. This chiral GSO projection gives rise to a supersymmetric type II string theory on (3.1) with at least $2^{\frac{d}{2}+1}$ spacetime supercharges. It is precisely this theory that appears as an equivalent description in the near-horizon geometry of fivebrane configurations in ten-dimensional type II string theory [38].

One can also impose a non-chiral GSO projection that gives rise to a type 0 string theory on (3.1). As their ten-dimensional non-supersymmetric cousins these theories possess a closed string tachyon, hence they are perturbatively unstable. Our objective here is to find an orientifold that projects out the tachyon and leads to a type $0'$ theory. We will focus on two specific examples with trivial \mathcal{M} and Γ : one at $d = 4$ and another one at $d = 6$. The possibility of generalizations will be discussed in the concluding section. The non-trivial part of the exercise is to identify the appropriate orientifold in $\mathcal{N} = 2$ Liouville

⁴In what follows daggers † will be used to denote complex conjugation in spacetime and bars right-moving quantities on the worldsheet.

theory and to analyze its properties. Orientifolds in $\mathcal{N} = 2$ Liouville theory have been constructed recently in [39].

Concluding this short introduction we would like to point out that stable non-supersymmetric vacua can be constructed in theories of the type (3.1) with appropriate deformations of type II string theory on (3.1). One such deformation arises when one changes the asymptotic radius of φ . Usually in this case a closed string tachyon appears infinitesimally close to the supersymmetric point, although there are some interesting exceptions [40]. If some of the flat $\mathbb{R}^{d-1,1}$ directions are compact, then the asymptotic background at infinity involves a higher dimensional torus. There is a certain subset of Kähler/complex structure deformations of this torus that breaks spacetime supersymmetry, but does not generate any closed string tachyons [41].

3.1 A few words on parities in $\mathcal{N} = 2$ Liouville theory

Before delving into specific examples, we would like to prepare the ground with a few words on certain parities of the $\mathcal{N} = 2$ Liouville theory. A general analysis of the orientifolds of the $\mathcal{N} = 2$ Liouville theory and its mirror dual supersymmetric $SL(2, \mathbb{R})/U(1)$ CFT can be found in [39].

In a two dimensional QFT with $\mathcal{N} = (2, 2)$ supersymmetry there are two basic types of parities that reverse the worldsheet chirality (exchanging the left- and right-movers) while preserving the holomorphy of the $\mathcal{N} = 2$ supersymmetry. They are known as A- and B-type and the details of their definition can be found in [42, 39]. A distinguishing feature of these parities in the case of the $\mathcal{N} = 2$ Liouville theory is the following. A-type parities give orientifolds that are localized in the direction φ , whereas B-type parities give orientifolds that are extended along φ . In the nomenclature of [39] the basic worldsheet parities are Ω_A and Ω_B . Ω_A acts on the $\mathcal{N} = 2$ Liouville worldsheet fields as

$$\Omega_A : \quad \Phi(z, \bar{z}) \rightarrow \Phi^\dagger(\bar{z}, z), \quad \psi \rightarrow \bar{\psi}^\dagger, \quad \bar{\psi} \rightarrow \psi^\dagger \quad (3.4)$$

and Ω_B as

$$\Omega_B : \quad \Phi(z, \bar{z}) \rightarrow \Phi(\bar{z}, z), \quad \psi \rightarrow \bar{\psi}, \quad \bar{\psi} \rightarrow \psi. \quad (3.5)$$

The B-parity is related to the standard worldsheet parity Ω , that appears in the previous section, by $\Omega_B = (-)^{\bar{F}}\Omega$.

We will be interested in parities that project out the tachyonic NSNS ground state, but retain the $\mathcal{N} = 2$ Liouville interaction that appears in (3.2). The A-type parity Ω_A exchanges the two superpotential terms in (3.2) and projects μ onto real values. However, for the purposes of this work only B-type orientifolds of $\mathcal{N} = 2$ Liouville will be relevant. As explained in [39], with B-type orientifolds there are two obvious candidates for the construction of type 0'B vacua. These are based on the $\mathcal{N} = 2$ Liouville parities

$$\mathcal{P} = s_\varphi \Omega_B, \quad \tilde{\mathcal{P}} = e^{\pi i \bar{F}} e^{-i\pi \bar{J}} \Omega_B, \quad (3.6)$$

where s_φ is the shift (2.4) acting on φ and \bar{J} is the right-moving $U(1)_R$ current. In the asymptotic weak coupling region, where the $U(1)_R$ currents J, \bar{J} take the simple form

$$J = F + i\sqrt{2}\partial\varphi, \quad \bar{J} = \bar{F} + i\sqrt{2}\bar{\partial}\varphi, \quad (3.7)$$

we can rewrite $\tilde{\mathcal{P}}$ as

$$\tilde{\mathcal{P}} = e^{-i\sqrt{2}\pi\bar{p}_\varphi}\Omega_B . \tag{3.8}$$

where \bar{p}_φ is the right-moving φ -momentum. Both \mathcal{P} and $\tilde{\mathcal{P}}$ project out the NSNS ground state, commute with the $\mathcal{N} = 2$ Liouville interaction and give space-filling orientifolds. In the ensuing, we will focus on the former parity which is geometric and has a clearer spacetime interpretation.

3.2 Type 0'B strings in four dimensions

In this subsection we consider a special case of (3.1) with $d = 4$, \mathcal{M} trivial and Γ the identity. In other words, we want to consider type 0B string theory on

$$\mathbb{R}^{3,1} \times \mathcal{N} = 2 \text{ Liouville} . \tag{3.9}$$

In this case, $\mathcal{N} = 2$ Liouville theory has a background charge $\mathcal{Q} = \sqrt{2}$, i.e. $k = 1$ in (3.2). Extrapolating the arguments of [38] to type 0 string theory, it is natural to conjecture that type 0B string theory on (3.9) provides in a double scaling limit a holographic description of the Little String Theory that lives on two orthogonal NS5-branes in ten-dimensional type 0B string theory. However, as the background is unstable the meaning of this correspondence is unclear.

The type 0B torus partition function of string theory on (3.9) can be found in [43, 44, 18]. Here we will be interested only in the leading piece of the torus partition function associated to the weakly coupled asymptotic linear dilaton region. It receives contributions only from the continuous representations of $\mathcal{N} = 2$ Liouville theory and is proportional to the infinite volume of the target space. In terms of the continuous representation characters, defined for generic k as

$$ch_c(p, m; \tau) \begin{bmatrix} a \\ b \end{bmatrix} = q^{\frac{p^2+m^2}{k}} \frac{\vartheta \begin{bmatrix} a \\ b \end{bmatrix}(\tau)}{\eta^3(\tau)} , \tag{3.10}$$

the type 0B torus partition function on (3.9) reads

$$\begin{aligned} \mathcal{T} = & \frac{\mathcal{V}}{2} \int_{\mathcal{F}} \frac{d\tau d\bar{\tau}}{4\tau_2} \sum_{a,b \in \mathbb{Z}_2} \frac{\vartheta \begin{bmatrix} a \\ b \end{bmatrix}(\tau) \vartheta \begin{bmatrix} a \\ b \end{bmatrix}(\bar{\tau})}{(8\pi^2\tau_2)^2 \eta^3(\tau) \eta^3(\bar{\tau})} \times \\ & \times \sum_{n,w \in \mathbb{Z}} \int_0^\infty dp \ ch_c\left(p, \frac{n+w}{2}; \tau\right) \begin{bmatrix} a \\ b \end{bmatrix} ch_c\left(p, \frac{n-w}{2}; \bar{\tau}\right) \begin{bmatrix} a \\ b \end{bmatrix} . \end{aligned} \tag{3.11}$$

The quantum numbers n and w represent the momentum and winding around the angular coordinate φ of $\mathcal{N} = 2$ Liouville theory (the former is broken by the $\mathcal{N} = 2$ Liouville interaction in the strong coupling region). The lowest level spectrum of this theory can be found in table 1 of appendix A of [43]. It comprises of the graviton multiplet (G, B, ϕ) , a tachyon T , two non-negative mass squared RR scalars C_0, C'_0 and a two-form potential C_2 . All these modes appear with any momentum and winding (n, w) .

The worldsheet parity \mathcal{P} , eq. (3.6), projects out the tachyonic NSNS ground state with $(n, w) = (0, 0)$, but keeps all the combinations of tachyon zero-modes of the form

$|(n, w)\rangle - (-)^n |(n, -w)\rangle$, which are all massless or massive. In particular, for $w = 0$, \mathcal{P} keeps all the modes with odd momentum n . This is an appealing feature, since it allows for the $\mathcal{N} = 2$ Liouville potential – with $(n, w) = (\pm 1, 0)$ – to be invariant. Hence, the parity \mathcal{P} is well defined in $\mathcal{N} = 2$ Liouville theory beyond the asymptotic region. Furthermore, it was pointed out in [39] that under \mathcal{P} the cigar interaction

$$\delta\mathcal{S} = \mu_{\text{cigar}} \int d^2z d^4\theta e^{-\frac{\Phi+\Phi^\dagger}{\sqrt{2}}} \tag{3.12}$$

is invariant. This property is required for the non-perturbative consistency of the theory.

The asymptotic Klein bottle amplitude associated with $[\mathbb{R}^{3,1} \times \mathcal{N} = 2 \text{ Liouville}]/\mathcal{P}$ reads

$$\mathcal{K} = \frac{\mathcal{V}}{2} \int_0^\infty \frac{dt}{2t} \sum_{a=0,1} (-)^a \sum_{n \in \mathbb{Z}} \int_0^\infty dp (-)^{n+1} ch_c\left(p, \frac{n}{2}\right) \begin{bmatrix} a \\ 1 \end{bmatrix} (2it) \frac{\vartheta \begin{bmatrix} a \\ 1 \end{bmatrix} (2it)}{(8\pi^2 t)^2 \eta^3(2it)}. \tag{3.13}$$

Only zero winding states in the $\widetilde{\text{NSNS}}$ and $\widetilde{\text{RR}}$ sectors contribute to the trace. Adding the contributions from (3.11) and (3.13) we obtain, as advertised, a tachyon-free spectrum.

In general, $\mathcal{N} = 2$ Liouville theory contains not only continuous representations, but also discrete representations describing states localized near the strong coupling region of the theory. One may wonder whether some localized tachyon may show up in these sectors. Fortunately, the discrete representations appear in the range

$$\frac{1}{2} < j < \frac{k+1}{2}, \tag{3.14}$$

and satisfy the conditions $p_\varphi/\mathcal{Q} - j \in \mathbb{Z}$ and $\bar{p}_\varphi/\mathcal{Q} - j \in \mathbb{Z}$. Therefore none of them appears in the model at hand.

In spacetime, the worldsheet parity \mathcal{P} gives rise to a space-filling $O'5$ -plane. Hence, it is important to check if this orientifold has any massless tadpoles. Using the well-known S-modular transformation properties of the continuous characters and the theta-functions (see *e.g.* [39]) we obtain from (3.13) the Klein bottle amplitude in the crosscap channel

$$\tilde{\mathcal{K}} = -\frac{\mathcal{V}}{8(2\pi)^4} \int_0^\infty dl \sum_{a \in \mathbb{Z}_2} \frac{\vartheta \begin{bmatrix} 1 \\ a \end{bmatrix} (il)}{\eta^3(il)} \sum_{w \in 2\mathbb{Z}+1} ch_c\left(0, \frac{w}{2}; il\right) \begin{bmatrix} 1 \\ a \end{bmatrix}. \tag{3.15}$$

This result is similar to the one obtained for our $\mathbb{R}^{8,1} \times S^1$ model in critical string theory, eq. (2.8). Most notably, the orientifold sources again only odd winding modes in the RR sector. All these modes are massive and hence the $O'5$ -plane in question does not exhibit any massless tree-level tadpoles. Consequently, this is an example of a non-supersymmetric theory of unoriented bosonic closed strings which is fully stable at tree-level as it exhibits neither tachyons nor tadpoles.

In this subsection we did not construct the exact crosscap state associated with the parity \mathcal{P} in the background (3.9), but this can be done with the technology of [39]. In that paper, the crosscap state of a related A-type orientifold in the mirror $\text{SL}(2, \mathbb{R})/\text{U}(1)$ was derived. We expect similar qualitative results for the $O'5$ -plane presented here. For more details see the closely related exact crosscap state that appears at the end of the following subsection.

Comments on D-branes. D-branes in type IIB string theory on $\mathbb{R}^{3,1} \times \text{SL}(2, \mathbb{R})/\text{U}(1)$, made of B-type branes of $\text{SL}(2, \mathbb{R})/\text{U}(1)$ (equivalently A-type branes in type IIA string theory on (3.9)) were used in [18, 45, 46] to engineer the electric and magnetic descriptions of $\mathcal{N} = 1$ SQCD with N_c colours and N_f flavors. Metastable non-supersymmetric configurations were discussed in this context in [47]. D-brane configurations with orientifolds were analyzed in [48].

In this paper we consider a different theory, type 0B on $\mathbb{R}^{3,1} \times \mathcal{N} = 2$ Liouville in the presence of an $O'5$ orientifold plane. The spectrum of allowed D-branes in this theory contains the following possibilities (see [49] and references therein for a comprehensive analysis of D-branes in $\mathcal{N} = 2$ Liouville theory):

- (i) D-branes that are localized in the strong coupling region of the $\mathcal{N} = 2$ Liouville throat. These branes have A-type boundary conditions in the $\mathcal{N} = 2$ Liouville part of the worldsheet CFT and depending on the boundary conditions that are imposed on the extra four flat directions we may have unstable Dp -branes for $p = 1, 3$, where p refers to the number of space-like dimensions filled by the brane in $\mathbb{R}^{3,1}$, or dyonic Dp -branes with $p = 0, 2$.⁵ The latter exhibit a vanishing annulus amplitude. All the branes have vanishing Möbius strip amplitude with the $O'5$ -plane. Indeed, notice that these branes, as A-type boundary states in $\mathcal{N} = 2$ Liouville, are coherent states of momentum modes, whereas the $O'5$ -plane, as a B-type crosscap state, is a coherent state of odd winding modes.
- (ii) D-branes that are extended in the linear dilaton direction and are A-type in the $\mathcal{N} = 2$ Liouville throat. In this case, we can have unstable Dp -branes for any even $p \leq 4$, or dyonic Dp -branes with odd $p \leq 5$. As in case (i) all the branes here have a vanishing Möbius strip amplitude with the $O'5$ -plane.
- (iii) D-branes that are extended in the linear dilaton direction and are B-type in the $\mathcal{N} = 2$ Liouville throat. In this case, there are unstable Dp -branes for any even $p \leq 4$ and dyonic Dp -branes with odd $p \leq 5$. These branes have a non-vanishing Möbius strip amplitude.

To illustrate the salient features of the above list we consider a few representative examples. Let us denote the boundary states that describe the Dp -branes in case (i) as $|Dp\rangle_{\text{NS}}$, $|Dp\rangle_{\text{R}}$ (in the NSNS and RR sectors respectively). Using the notation of [18] for the $\mathcal{N} = 2$ Liouville part of the boundary states and the standard notation of [19] for the

⁵The word dyonic refers here to the standard pair of an electric and a magnetic boundary state of the type 0B theory. These branes do not have an open string tachyon on their worldvolume. Notice that because of the A-type boundary conditions in $\mathcal{N} = 2$ Liouville, one can have stable even-dimensional localized D-branes in the non-critical type 0B theory, unlike in ten-dimensions.

fermionic part, we can write $|Dp\rangle_{\text{NS}}$ in terms of (extended) Ishibashi states as⁶

$$|Dp\rangle_{\text{NS}} = \int_0^\infty dP \left[\Phi_{\text{NS}}(P, 0; +)(|P, 0; +\rangle_{\text{NSNS}} - |P, 0; -\rangle_{\text{NSNS}}) + \right. \tag{3.16}$$

$$\left. + \Phi_{\text{NS}}\left(P, \frac{1}{2}; -\right)(|P, \frac{1}{2}; +\rangle_{\text{NSNS}} + |P, \frac{1}{2}; -\rangle_{\text{NSNS}}) \right].$$

Using the known action of Ω , $(-)^{\bar{F}}$ and s_φ on the boundary states [19, 50]

$$\begin{aligned} \Omega|P, m; \eta\rangle_{\text{NSNS}} &= |P, m; \eta\rangle_{\text{NSNS}}, \\ \Omega|P, m; \eta\rangle_{\text{RR}} &= -(i\eta)^{2-p}|P, m; \eta\rangle_{\text{RR}}, \\ (-)^{\bar{F}}|P, m; \eta\rangle_{\text{NSNS}} &= -|P, m; -\eta\rangle_{\text{NSNS}}, \\ (-)^{\bar{F}}|P, m; \eta\rangle_{\text{RR}} &= |P, m; -\eta\rangle_{\text{RR}}, \\ s_\varphi|P, m; \eta\rangle_{\text{NSNS/RR}} &= (-)^{2m}|P, m; \eta\rangle_{\text{NSNS/RR}}, \\ m &= 0, \frac{1}{2}, \quad \eta = \pm \end{aligned} \tag{3.17}$$

we deduce that $|Dp\rangle_{\text{NS}}$ is invariant under $\mathcal{P} = s_\varphi(-)^{\bar{F}}\Omega$. This boundary state describes for odd p an unstable brane, which includes an open string tachyon in its spectrum. For even p this NSNS boundary state alone describes a Dp - $\bar{D}p$ pair as in ten dimensions. The wavefunctions $\Phi_{\text{NS}}(P, 0; +)$, $\Phi_{\text{NS}}(P, \frac{1}{2}; -)$ appear in a type II context in [18].

To obtain branes without open string tachyons one needs to add a RR sector contribution to the NSNS sector D-brane boundary state. This is possible only for even p as for odd p the resulting boundary state is not GSO invariant. This is an unfortunate feature for the construction of interesting four dimensional gauge theories in this non-supersymmetric setup (such theories would be realized on D3-branes at the strong coupling region of the $\mathcal{N} = 2$ Liouville throat). It should have been expected, however, since we are considering a type 0B theory and the only known branes with ‘‘Dirichlet’’ boundary conditions in the linear dilaton direction are the ones that are A-type in $\mathcal{N} = 2$ Liouville theory.

For even p we consider the generic linear superposition of Ishibashi states in the RR sector:⁷

$$|Dp\rangle_{\text{R}} = \int_0^\infty dP \left[\Phi_{\text{R}}(P, 0; +)(|P, 0; +\rangle_{\text{RR}} + \alpha|P, 0; -\rangle_{\text{RR}}) + \right. \tag{3.18}$$

$$\left. + \Phi_{\text{R}}\left(P, \frac{1}{2}; -\right)(|P, \frac{1}{2}; +\rangle_{\text{RR}} + \beta|P, \frac{1}{2}; -\rangle_{\text{RR}}) \right].$$

This RR boundary state is invariant under the parity \mathcal{P} iff

$$\alpha = i^{-p}, \quad \beta = -i^{-p}. \tag{3.19}$$

⁶We quickly remind the reader that the fermionic boundary states $|\eta\rangle_{\text{NSNS/RR}}$ exhibit the overlaps ${}_{\text{NSNS}}\langle\eta|\eta'\rangle_{\text{NSNS}} \sim \left[\frac{0}{1-\eta\eta'}\right]$, ${}_{\text{RR}}\langle\eta|\eta'\rangle_{\text{RR}} \sim \left[\frac{1}{1-\eta\eta'}\right]$, where the rhs denotes the sector $\left[\begin{smallmatrix} a \\ b \end{smallmatrix}\right]$ in which the standard fermionic characters appear.

⁷See [18] for the exact expressions of the R sector wavefunctions $\Phi_{\text{R}}(P, 0; +)$, $\Phi_{\text{R}}(P, \frac{1}{2}; -)$.

So we find for $p = 0, 2$ the \mathcal{P} -invariant boundary states $|Dp\rangle_{\text{NS}} \pm |Dp\rangle_R$ (the upper and lower sign refer to D-branes and anti-D-branes respectively). They are identical to the boundary states of the corresponding dyonic Dp -branes in the type 0B theory on (3.9) and give a vanishing annulus amplitude. We find that a stack of N such D-branes realize a non-supersymmetric gauge theory with $U(N)$ gauge group, an adjoint real scalar, as well as fermions in the symmetric and antisymmetric representations. This theory is a non-supersymmetric descendant of the three-dimensional $\mathcal{N} = 2$ SYM; the latter can be realized in type IIB string theory on (3.9) with the corresponding BPS D2-branes.⁸

The above exercise can be repeated *mutatis mutandis* for A-type D-branes of $\mathcal{N} = 2$ Liouville theory that are extended in the linear dilaton direction. With a few modifications similar results can also be obtained for extended D-branes that are B-type in $\mathcal{N} = 2$ Liouville theory (boundary states for such branes can be deduced easily from the boundary CFT analysis of [49]). In this case, there is a non-vanishing Möbius strip amplitude that can be determined with the use of the exact crosscap wavefunction that appears in the following subsection (appropriately adapted to the four dimensional case). We will not, however, pursue this calculation any further here.

3.3 Type 0'B strings in six dimensions

We now turn to an example with six dimensional Poincaré invariance that involves type 0B string theory on

$$\mathbb{R}^{5,1} \times \mathcal{N} = 2 \text{ Liouville.} \tag{3.20}$$

It describes the near-horizon limit of a pair of parallel fivebranes. This is a special case of the CHS background $\mathbb{R}^{5,1} \times \mathbb{R}_Q \times \text{SU}(2)_k$ (the near-horizon geometry of k parallel NS5-branes [28]), with $k = 2$. At this particular value of the level k , the supersymmetric $\text{SU}(2)$ WZW model contains only an $\text{SU}(2)_2 \times \text{SU}(2)_2$ algebra consisting of three worldsheet fermions (both left- and right-handed), as the bosonic WZW model becomes trivial. After the bosonization of the two complex fermions ψ^\pm and $\bar{\psi}^\pm$ (the superpartners of the $\text{SU}(2)$ currents J^\pm and \bar{J}^\pm) we obtain a $U(1)$ affine algebra at level $k = 2$. This observation allows us to rewrite the torus amplitude for the type 0B theory as:

$$\begin{aligned} \mathcal{T} = & \frac{\mathcal{V}}{2} \int \frac{d\tau d\bar{\tau}}{4\tau_2} \frac{1}{(8\pi^2\tau_2)^3} \sum_{a,b \in \mathbb{Z}_2} \frac{|\vartheta \begin{bmatrix} a \\ b \end{bmatrix}|^4}{|\eta|^{12}} \times \\ & \times \int_0^\infty dp \sum_{r, \bar{r} \in \mathbb{Z}} (-)^{b(r-\bar{r})} ch_c(p, r + \frac{a}{2}; \tau) \begin{bmatrix} a \\ b \end{bmatrix} \bar{ch}_c(p, \bar{r} + \frac{a}{2}; \bar{\tau}) \begin{bmatrix} a \\ b \end{bmatrix}. \end{aligned} \tag{3.21}$$

We identify the momentum n and the winding number w in the $\mathcal{N} = 2$ Liouville as

$$n = \frac{r + \bar{r} + a}{2}, \quad w = r - \bar{r}. \tag{3.22}$$

⁸Flavors can be added to both of these theories with the addition of the appropriate D4-branes to obtain three dimensional $\mathcal{N} = 2$ SQCD or its non-supersymmetric descendant. It would be interesting to consider Seiberg duality for these gauge theories (especially the non-supersymmetric one) in the setup of the non-critical string generalizing the arguments of [46, 48].

Fractional momenta are allowed since the generalized diagonal projection acts as a \mathbb{Z}_2 winding shift orbifold of the transverse direction φ in $\mathcal{N} = 2$ Liouville.⁹

As the torus partition function (3.21) is very closely related to that of the critical ten-dimensional type 0B superstring, one can write down a Klein bottle amplitude for the 0'B theory which is similar to that of [1–3] (modulo two η -functions). In terms of the $\mathcal{N} = 2$ Liouville continuous characters (3.10) we obtain

$$\mathcal{K} = -\frac{\mathcal{V}}{2} \int_0^\infty \frac{dt}{2t} \frac{1}{(8\pi^2 t)^3} \sum_{a \in \mathbb{Z}_2} (-)^a \frac{\vartheta^2 \left[\begin{smallmatrix} a \\ 1 \end{smallmatrix} \right] (2it)}{\eta^6(2it)} \int_0^\infty dp \sum_{n \in \mathbb{Z}} e^{i\pi(n+a/2)} ch_c(p, n + a/2; 2it) \begin{bmatrix} a \\ 1 \end{bmatrix}. \quad (3.23)$$

Alternatively, one can derive this amplitude from the worldsheet parity \mathcal{P} in (3.6). This justifies *a posteriori* the choice of a B-parity of $\mathcal{N} = 2$ Liouville in order to construct a type 0' string theory in the non-critical background (3.20). As the theory involves half-integral momentum one may wonder whether this parity is involutive. However, the potentially non-trivial phases occur in the RR sector (*i.e.* $a = 1$), where the fermions contribute to \mathcal{P}^2 as $\exp 3i\pi/2$, hence we deduce that the parity is indeed involutive.

Adding the torus and Klein bottle amplitudes (3.21) and (3.23) respectively, we obtain as anticipated a tachyon-free continuous spectrum.¹⁰ As before there may be localized tachyons in the discrete representations. Using (3.14) one finds that the only physical discrete states have $j = 1$. The lowest dimension operator is the $\mathcal{N} = 2$ Liouville chiral primary with $n = 1$ and $w = 0$, of dimension $L_0 = j/k = 1/2$. Hence it gives a massless state in spacetime, which is none other than the momentum condensate in the $\mathcal{N} = 2$ Liouville action, eq. (3.2).

The six dimensional type 0'B theory is also free of massless tadpoles. In the transverse channel, the Klein bottle amplitude (3.23) reads:

$$\tilde{\mathcal{K}} = -\frac{\mathcal{V}}{8(2\pi)^6} \int_0^\infty dl \sum_{a \in \mathbb{Z}_2} (-)^a \frac{\vartheta^2 \left[\begin{smallmatrix} 1 \\ a \end{smallmatrix} \right] (il)}{\eta^6(il)} \sum_{w \in 2\mathbb{Z}+1} (-)^{\frac{a(w-1)}{2}} ch_c \left(0, \frac{w}{2}; il \right) \begin{bmatrix} 1 \\ a \end{bmatrix} \quad (3.24)$$

coupling only to odd winding states in the RR sector. In the $l \rightarrow \infty$ limit, the dominant contribution to this amplitude comes from states with unit winding $w = \pm 1$ and $\tilde{\mathcal{K}}$ behaves like

$$\tilde{\mathcal{K}} \sim -\frac{\mathcal{V}}{8(2\pi)^6} \int_0^\infty dl e^{-\frac{\pi l}{4}} \left[1 + \mathcal{O}(e^{-2\pi l}) \right]. \quad (3.25)$$

We conclude that the RR tadpoles are again massive. As in the previous four dimensional example, this is due to the "universal" mass shift $m_{\min}^2 = Q^2/4$ in linear dilaton backgrounds (for the continuous representations).¹¹ Hence, we obtain another example of

⁹It should be noted that there is another possible type 0B modular invariant, where the \mathbb{Z}_2 orbifold acts only on the fermions and not on the $U(1)_2$ boson φ . Both theories are consistent (both exhibit mutual locality of operators), but only the first one describes the near horizon geometry of two parallel fivebranes [43]. In what follows, we will consider only this case.

¹⁰It is interesting to remark that, without the non-trivial action of the GSO projection on the compact boson φ of $\mathcal{N} = 2$ Liouville (in the type 0B model we consider), one would have winding tachyons surviving the orientifold projection.

¹¹This property of linear dilaton backgrounds was also used in [51] as an infrared regulator in closed string one-loop computations.

a closed unoriented theory which is perturbatively stable, tadpole-free and does not require the addition of open string sectors for consistency.

One can construct various D-branes in the six-dimensional non-critical type 0'B theory, along the lines of our analysis of the four-dimensional model. There is however an important difference. Because of the non-trivial GSO projection, and the enlarged $\mathcal{N} = (4, 4)$ superconformal symmetry of $\mathcal{N} = 2$ Liouville theory with $\mathcal{Q} = 1$, it is actually possible to evade the constraints found in four dimensions and obtain stable D-branes filling $\mathbb{R}^{3,1}$, localized in the linear dilaton direction. This allows to engineer four-dimensional, non-supersymmetric gauge theories (with fermions). We postpone the detailed study of the properties of these theories to future work.

The exact crosscap state. Crosscap states in $\mathcal{N} = 2$ Liouville theory and its mirror dual $SL(2, \mathbb{R})/U(1)$ supersymmetric coset were constructed in [39]. Using the results of that paper (more specifically the wavefunction for the B-type parity of $\mathcal{N} = 2$ Liouville), we obtain the following candidate for the type 0'B crosscap wavefunction:

$$\Psi(p^\mu, P, w) \begin{bmatrix} b \\ a \end{bmatrix} \propto e^{\frac{i\pi}{2}(1-a) + \frac{i\pi a(w-1)}{4}} \delta_{b,1 \bmod 2} \delta_{w,1 \bmod 2} \delta^6(p^\mu) \times \tag{3.26}$$

$$\times 2^{-iP} \cosh \frac{\pi P}{2} \frac{\Gamma(1-iP)\Gamma(-2iP)}{\Gamma(\frac{1}{2}-iP+\frac{w-1}{2})\Gamma(\frac{1}{2}-iP-\frac{w-1}{2})},$$

up to a normalization constant that is fixed by channel duality. The labels of the wavefunction Ψ on the rhs of this equation denote in an obvious fashion the quantum numbers of the closed string vertex operators whose one-point function on the \mathbb{RP}_2 we are computing. A similar expression holds for the orientifold of the four dimensional non-critical string theory considered in the previous subsection.

4. Discussion

In this note we presented two tachyon-free examples of type 0'B closed string theories without massless tree-level tadpoles on backgrounds with non-trivial dilaton profiles. We would like to conclude with a list of interesting questions and related comments.

- (i) *Fivebranes, double-scaling limits and LST's in type 0.* Backgrounds of the form (3.1) can be embedded naturally in ten-dimensional type II string theories as backgrounds that describe string theory dynamics in the near-horizon region of fivebrane configurations or in the vicinity of Calabi-Yau singularities in a double scaling limit [38]. The simplest example, that we discussed in the previous section, corresponds to parallel fivebranes. As reviewed there, the six dimensional non-critical string arises as a special, degenerate case within a family of *critical* string backgrounds describing the near-horizon limit of k parallel fivebranes.¹² A natural question concerns the embedding of type 0'B non-critical string theories in ten dimensions with an appropriate

¹²Similarly type II string theory on $\mathbb{R}^{3,1} \times \mathcal{N} = 2$ Liouville describes string propagation in the near-horizon region of two orthogonal NS5-branes, or in the vicinity of the conifold singularity.

configuration of fivebranes and orientifolds. Clearly, the ten-dimensional theory cannot be type 0'B, because in that theory the NSNS two-form is projected out and the theory cannot accommodate any NS5-branes.¹³

An important clue comes from the classification of orientifolds in the SU(2) WZW model, see e.g. [52]. Consistent orientifolds in this theory by definition preserve the Wess-Zumino term and can be either a pair of antipodal O0-planes or an equatorial O2-plane. In order to generalize the six-dimensional non-critical type 0'B theory to arbitrary values of k , one can build a type 0A model on the general CHS background $\mathbb{R}^{5,1} \times \mathbb{R}_Q \times \text{SU}(2)_k$ with an orientifold that is space-filling in $\mathbb{R}^{5,1} \times \mathbb{R}_Q$ and has the geometry of a pair of O0-planes in $\text{SU}(2)_k$. This setup is such that it can be interpreted as the near-horizon limit of k NS5-branes at the origin of the transverse space $x^{6,7,8,9}$ in the unoriented string theory $0A/(\mathcal{I}_{678}(-)^{\bar{F}}\Omega)$.¹⁴ Indeed, in spherical coordinates for the transverse space, the orientifold is extended along the radial (linear dilaton) direction and intersects the three-sphere at two antipodal points.

For k even one can spread the fivebranes evenly on a circle in the (x^6, x^7) plane in the presence of the orientifold.¹⁵ This amounts to adding the $\mathcal{N} = 2$ Liouville interaction on the worldsheet theory of $\mathbb{R}_Q \times \text{SU}(2)_k$, see the last two terms in eq. (3.2). By T-duality – along the angular coordinate in the (x^8, x^9) plane – one gets type 0B in $[\mathcal{N} = 2 \text{ Liouville} \times \text{SU}(2)_k / \text{U}(1)] / \mathbb{Z}_k$. In this mirror background the orientifold is made of the B-type O2-plane of $\mathcal{N} = 2$ Liouville [39], and the point-like B-type O0 orientifold of $\text{SU}(2)/\text{U}(1)$ that exists for k even [42]. The Klein bottle amplitude for the resulting model reads:

$$\begin{aligned} \mathcal{K} = & -\frac{\mathcal{V}}{2} \int \frac{dt}{2t} \sum_a (-)^a \frac{1}{(8\pi^2 t)^3} \frac{\vartheta^2 \left[\begin{smallmatrix} a \\ 1 \end{smallmatrix} \right]}{\eta^6} \times \\ & \times \int_0^\infty dp \sum_{m \in \mathbb{Z}_{2k}, w \in \mathbb{Z}} ch_c \left(p, \frac{m}{2} + kw; 2it \right) \left[\begin{smallmatrix} a \\ 1 \end{smallmatrix} \right] \sum_{2j=0}^{k-2} (-)^m C_m^j \left[\begin{smallmatrix} a \\ 1 \end{smallmatrix} \right] (2it) \end{aligned} \quad (4.1)$$

where $C_m^j \left[\begin{smallmatrix} a \\ b \end{smallmatrix} \right]$ are characters of the super-coset $\text{SU}(2)/\text{U}(1)$. At $k = 2$, the supercoset $\text{SU}(2)/\text{U}(1)$ contains only the identity. Hence, the following relation holds

$$\sum_j C_m^j \left[\begin{smallmatrix} a \\ 1 \end{smallmatrix} \right] = e^{-\frac{i\pi a}{2}} [\delta_{m,a \bmod 4} - \delta_{m,a+2 \bmod 4}] \quad \text{for } k = 2. \quad (4.2)$$

Consequently, one finds that the Klein bottle amplitude in the background of two fivebranes, eq. (4.1), becomes precisely the same as that of the type 0'B non-critical strings in six dimensions, eq. (3.23).¹⁶ Notice that the (asymptotic) translation symmetry $\varphi \rightarrow \varphi + \lambda$ in $\mathcal{N} = 2$ Liouville corresponds to rotations in the (x^6, x^7) plane

¹³It is worth noticing that in the non-critical type 0'B theories of the previous section certain modes of the NSNS two-form are not projected out and remain in the physical spectrum.

¹⁴A type IIA orientifold of the CHS background with the same geometry was studied in [53, 54].

¹⁵For k odd the distribution of fivebranes is not symmetric under \mathcal{I}_{678} .

¹⁶Note that if one had chosen instead an O2-plane of $\text{SU}(2)$ – giving an O2-plane of $\text{SU}(2)/\text{U}(1)$ – there would be an extra phase $(-)^a$ in (3.23). This would reverse the orientifold projection in the RR sector.

where the two NS5-branes are separated,¹⁷ while the winding symmetry $\tilde{\varphi} \rightarrow \tilde{\varphi} + \tilde{\lambda}$ corresponds to rotations in the (x^8, x^9) plane. As the fixed point of s_φ is the origin of the (x^6, x^7) plane, and the crosscap has couplings to winding modes, this is in accordance with the conclusion that the geometry of the orientifold is $x^{6,7,8} = 0$, *i.e.* that it is an $O'6$ -plane in the fivebrane geometry. These observations can be used as further evidence in favor of the crosscap appearing in eq. (3.26). Moreover, one can add in type 0A non-tachyonic D4-branes stretched between the two parallel NS5-branes, along x^6 . In the presence of the $O'6$ plane, this configuration leads to a non-supersymmetric four-dimensional gauge theory as mentioned in the previous section.

One may wonder whether generically type 0A string theory in the near-horizon geometry of k fivebranes with an $O'6$ orientifold is also perturbatively stable. In flat space, the orientifold $0A/(\mathcal{I}_{678}(-)^{\bar{F}}\Omega)$ contains a "bulk" tachyon corresponding roughly speaking to closed strings away from the $O'6$ -plane. In general, adding k NS5-branes and going to the near-horizon limit – the CHS background – does not help. A tachyon appears in the representation $j = 1/2$ of $SU(2)_{k,L} \times SU(2)_{k,R}$ (as $2j + m = 0 \pmod{2}$ in the NS sector).¹⁸ At $k = 2$ only the "s-wave" (*i.e.* $j = 0$) remains in the physical spectrum and therefore the tachyon is gone. This shows that the tachyon-free six dimensional type $0'$ non-critical string on $\mathbb{R}^{5,1} \times \mathcal{N} = 2$ Liouville is an exceptional case within the class of backgrounds that appear in the near-horizon geometry of k parallel fivebranes.

The four-dimensional non-critical string is also expected to be related to a type 0 critical string theory in the background of two orthogonal fivebranes with some O' -plane. However, as this model does not belong to a family of critical backgrounds with a similar interpretation (*i.e.* more than two orthogonal fivebranes) there is no direct way to prove this statement.

One can use the non-critical type $0'$ vacua of this note as the holographic definition (in the sense of [27, 38]) of a new kind of "bosonic" Little String Theories. It would be interesting to understand certain aspects of these dual theories, for instance, their low-energy dynamics. It would be also nice to elaborate further on other four-dimensional non-critical strings of the form (3.1) with non-trivial CFT \mathcal{M} .¹⁹

- (ii) *Comments on asymptotic supersymmetry and holography.* An interesting general question in string theory is the following: how badly can supersymmetry be broken in a stable vacuum? In string theory this question is important in the context of the general problem of supersymmetry breaking and vacuum selection, the problem of the cosmological constant and for holography in non-supersymmetric situations. For

¹⁷Only a \mathbb{Z}_2 subgroup is preserved by the $\mathcal{N} = 2$ Liouville interaction in the GSO-projected theory (3.21), as the fivebranes break the rotational symmetry.

¹⁸For $k > 2$ there may be also localized tachyons in the spectrum, *i.e.* made with the discrete representations of $\mathcal{N} = 2$ Liouville.

¹⁹With $SU(2)_k/U(1)$, giving the CHS background, we exhausted all the possibilities in six dimensions up to discrete identifications.

instance, strongly coupled QCD in the limit of a large number of colors is believed to be a stable theory with an exponential density of weakly interacting hadrons [55, 56]. This theory is also expected on general grounds [57] to have a dual formulation as a string theory (although the precise string theory with this property is not presently known). This example would suggest that there are stable weakly interacting vacua of string theory where supersymmetry is altogether absent and the spectrum comprises only of bosons. Indeed, a number of such theories were discussed in this note, although we only presented examples with unoriented closed strings. As we now review, there is a deep reason for this feature.

At weak coupling one can make a number of interesting general statements concerning the spectrum of stable string theories. At the level of the torus amplitude there can be only one kind of divergence (in more than two dimensions), an IR divergence due to a closed string tachyon. Then, under very mild assumptions, one can argue on the basis of worldsheet modular invariance [10, 37] that a tachyon divergence is directly related to an exponential mismatch between the asymptotic high energy density of bosons and fermions. When the tachyon is absent, the asymptotic high energy density of bosons will in general balance the number of fermions up to a two-dimensional field theoretic density of states. This asymptotic cancellation between the number of bosons and fermions has been dubbed *asymptotic supersymmetry* (or misaligned supersymmetry in [11, 12]). Hence, the theorem of [10] places a non-trivial constraint, the constraint of asymptotic supersymmetry, on how badly non-supersymmetric a stable theory of oriented strings can be at weak coupling.²⁰ It should be emphasized that the criterion of asymptotic supersymmetry is a purely stringy effect that is not visible in the low energy supergravity description of the theory.

In this paper we discussed a number of perturbative string theory vacua, where asymptotic supersymmetry is altogether absent, but the theory is nevertheless stable at tree level. The extra ingredient that allowed us to evade the general one-loop arguments of [10] is the contribution of the Klein bottle to the closed string one-loop vacuum amplitude, which in certain cases can project out the would-be closed string tachyon. Hence, we may conclude that asymptotic supersymmetry in the bulk is not, in general, a necessary condition for tree level stability in string theory. Nevertheless, we would like to argue here that the stability of closed strings requires asymptotic supersymmetry in a slightly different form.

Using general arguments based on worldsheet duality, one can argue for a connection between asymptotic supersymmetry in the open string sectors associated with the various D-branes accommodated by a given string theory and IR instabilities in the bulk [13]. More specifically, an open string theory without asymptotic supersymmetry is necessarily coupled to a *closed* string tachyon, which one can argue cannot be decoupled even in the low energy limit. It should be noted that open string tachyons do not partake in this connection with the asymptotic high energy density of open string states, instead they signal the existence

²⁰Since string theory on RR backgrounds is much less understood, it is not immediately clear if this theorem can be evaded in this case.

of lower energy configurations for the D-branes in question. We would like to propose that when a theory has a non-trivial spectrum of D-branes, asymptotic supersymmetry on the branes is a more fundamental property than asymptotic supersymmetry in the bulk as far as the stability of the theory in the bulk is concerned.²¹ A natural general conjecture for which there is no known counterexample is the following:²²

Conjecture: for theories that do not admit open strings, like the heterotic string, asymptotic supersymmetry in the bulk is necessary and sufficient for tree-level stability by the standard arguments of [10]. For theories with oriented or unoriented strings in the bulk that admit open strings, the closed string spectrum is tachyon free if and only if all the open string sectors associated with the allowed D-branes of the theory exhibit asymptotic supersymmetry.

Strong evidence for the validity of this conjecture arises from the combined arguments of [10, 13].

The above observations can have indirect consequences for the engineering of non-supersymmetric gauge theories and holography in string theory. Suppose we want to engineer a gauge theory as the low energy limit of an open string theory on a stack of D-branes in a background without closed string tachyons. Then, according to the above conjecture the open string theory on the D-branes will necessarily exhibit asymptotic supersymmetry.²³ This is a property of the high-energy degrees of freedom of string theory, so it is difficult to see precisely how this property will affect the low-energy field-theoretic degrees of freedom. At least one can deduce with certainty that the full open string theory spectrum on the branes will necessarily include both bosons and fermions. In the usual fermionic string this would imply the presence of both NS and R sectors. Hence, if we are looking for a setup that realizes a purely bosonic gauge theory (for example four dimensional bosonic Yang-Mills theory), somehow we have to find a way to give a (stringy) mass to the fermions in the R sectors and it is not immediately clear if and how this could be achieved.²⁴ A few examples that verify this difficulty include the dyonic branes in type 0B string theory in ten dimensions and the D-branes in type 0' theory in ten dimensions or in non-critical dimensions (see sections 2 and 3 above). It would appear that perturbative stability in closed string theory comes with a set of constraining conditions and not everything goes as one might imagine from low energy effective action intuitions. It would be very interesting to get a better handle on this set of constraining conditions (the above conjecture is an example of such constraints).

²¹A similar phenomenon has been observed in matrix theory, where asymptotic supersymmetry seems to be a crucial requirement for locality and cluster decomposition [58].

²²VN would like to thank Adi Armoni for a discussion on this point.

²³Even in backgrounds with closed string tachyons we should consider D-brane configurations with an asymptotically supersymmetric open string spectrum. Otherwise it is impossible to decouple open strings from the closed string tachyon in the usual low-energy decoupling limits [13].

²⁴Of course, one could consider D-brane setups where the low energy theory is a gauge theory with both bosons and fermions. In this context one can give an arbitrary bare mass to the fermions and flow to the IR, where presumably one obtains a purely bosonic theory. This is not what we have in mind here. What we are looking for are D-brane setups that realize a bosonic gauge theory at all scales after sending $\alpha' \rightarrow 0$.

Acknowledgments

We would like to thank Carlo Angelantonj, Adi Armoni, Emilian Dudas and David Kutasov for useful discussions and comments. VN acknowledges partial financial support by the EU under the contracts MEXT-CT-2003-509661, MRTN-CT-2004-005104 and MRTN-CT-2004-503369.

References

- [1] M. Bianchi and A. Sagnotti, *On the systematics of open string theories*, *Phys. Lett.* **B 247** (1990) 517.
- [2] A. Sagnotti, *Some properties of open string theories*, [hep-th/9509080](#).
- [3] A. Sagnotti, *Surprises in open-string perturbation theory*, *Nucl. Phys.* **56B** (*Proc. Suppl.*) (1997) 332 [[hep-th/9702093](#)].
- [4] W. Fischler and L. Susskind, *Dilaton tadpoles, string condensates and scale invariance*, *Phys. Lett.* **B 171** (1986) 383.
- [5] W. Fischler and L. Susskind, *Dilaton tadpoles, string condensates and scale invariance. 2*, *Phys. Lett.* **B 173** (1986) 262.
- [6] S.R. Das and S.-J. Rey, *Dilaton condensates and loop effects in open and closed bosonic strings*, *Phys. Lett.* **B 186** (1987) 328.
- [7] E. Dudas and J. Mourad, *Brane solutions in strings with broken supersymmetry and dilaton tadpoles*, *Phys. Lett.* **B 486** (2000) 172 [[hep-th/0004165](#)].
- [8] R. Blumenhagen and A. Font, *Dilaton tadpoles, warped geometries and large extra dimensions for non-supersymmetric strings*, *Nucl. Phys.* **B 599** (2001) 241 [[hep-th/0011269](#)].
- [9] C. Angelantonj and E. Dudas, *Metastable string vacua*, [arXiv:0704.2553](#).
- [10] D. Kutasov and N. Seiberg, *Number of degrees of freedom, density of states and tachyons in string theory and CFT*, *Nucl. Phys.* **B 358** (1991) 600.
- [11] K.R. Dienes, *Modular invariance, finiteness and misaligned supersymmetry: new constraints on the numbers of physical string states*, *Nucl. Phys.* **B 429** (1994) 533 [[hep-th/9402006](#)].
- [12] K.R. Dienes, M. Moshe and R.C. Myers, *String theory, misaligned supersymmetry and the supertrace constraints*, *Phys. Rev. Lett.* **74** (1995) 4767 [[hep-th/9503055](#)].
- [13] V. Niarchos, *Density of states and tachyons in open and closed string theory*, *JHEP* **06** (2001) 048 [[hep-th/0010154](#)].
- [14] I.R. Klebanov and A.A. Tseytlin, *D-branes and dual gauge theories in type 0 strings*, *Nucl. Phys.* **B 546** (1999) 155 [[hep-th/9811035](#)].
- [15] C. Angelantonj and A. Armoni, *Non-tachyonic type 0B orientifolds, non-supersymmetric gauge theories and cosmological RG flow*, *Nucl. Phys.* **B 578** (2000) 239 [[hep-th/9912257](#)].
- [16] R. Blumenhagen, A. Font and D. Lüst, *Non-supersymmetric gauge theories from D-branes in type 0 string theory*, *Nucl. Phys.* **B 560** (1999) 66 [[hep-th/9906101](#)].
- [17] P. Di Vecchia, A. Liccardo, R. Marotta and F. Pezzella, *Brane-inspired orientifold field theories*, *JHEP* **09** (2004) 050 [[hep-th/0407038](#)].

- [18] A. Fotopoulos, V. Niarchos and N. Prezas, *D-branes and SQCD in non-critical superstring theory*, *JHEP* **10** (2005) 081 [[hep-th/0504010](#)].
- [19] M.R. Gaberdiel, *Lectures on non-BPS Dirichlet branes*, *Class. and Quant. Grav.* **17** (2000) 3483 [[hep-th/0005029](#)].
- [20] E. Dudas, G. Pradisi, M. Nicolosi and A. Sagnotti, *On tadpoles and vacuum redefinitions in string theory*, *Nucl. Phys.* **B 708** (2005) 3 [[hep-th/0410101](#)].
- [21] C. Angelantonj, *Non-tachyonic open descendants of the 0B string theory*, *Phys. Lett.* **B 444** (1998) 309 [[hep-th/9810214](#)].
- [22] R. Blumenhagen, A. Font and D. Lüst, *Tachyon-free orientifolds of type 0B strings in various dimensions*, *Nucl. Phys.* **B 558** (1999) 159 [[hep-th/9904069](#)].
- [23] R. Blumenhagen and A. Kumar, *A note on orientifolds and dualities of type 0B string theory*, *Phys. Lett.* **B 464** (1999) 46 [[hep-th/9906234](#)].
- [24] K. Forger, *On non-tachyonic $z(n) \times z(m)$ orientifolds of type 0B string theory*, *Phys. Lett.* **B 469** (1999) 113 [[hep-th/9909010](#)].
- [25] C. Angelantonj and A. Sagnotti, *Open strings*, *Phys. Rept.* **371** (2002) 1 [[hep-th/0204089](#)].
- [26] I. Brunner and K. Hori, *Notes on orientifolds of rational conformal field theories*, *JHEP* **07** (2004) 023 [[hep-th/0208141](#)].
- [27] O. Aharony, M. Berkooz, D. Kutasov and N. Seiberg, *Linear dilatons, NS5-branes and holography*, *JHEP* **10** (1998) 004 [[hep-th/9808149](#)].
- [28] C.G. Callan Jr., J.A. Harvey and A. Strominger, *Supersymmetric string solitons*, [hep-th/9112030](#).
- [29] A. Giveon and D. Kutasov, *Little string theory in a double scaling limit*, *JHEP* **10** (1999) 034 [[hep-th/9909110](#)].
- [30] N. Seiberg, *New theories in six dimensions and matrix description of M-theory on T^5 and T^5/\mathbb{Z}_2* , *Phys. Lett.* **B 408** (1997) 98 [[hep-th/9705221](#)].
- [31] T. Takayanagi and N. Toumbas, *A matrix model dual of type 0B string theory in two dimensions*, *JHEP* **07** (2003) 064 [[hep-th/0307083](#)].
- [32] M.R. Douglas et al., *A new hat for the $c = 1$ matrix model*, [hep-th/0307195](#).
- [33] J. Gomis and A. Kapustin, *Two-dimensional unoriented strings and matrix models*, *JHEP* **06** (2004) 002 [[hep-th/0310195](#)].
- [34] O. Bergman and S. Hirano, *The cap in the hat: unoriented 2D strings and matrix(-vector) models*, *JHEP* **01** (2004) 043 [[hep-th/0311068](#)].
- [35] Y. Nakayama, *Tadpole cancellation in unoriented Liouville theory*, *JHEP* **11** (2003) 017 [[hep-th/0309063](#)].
- [36] D. Kutasov and N. Seiberg, *Noncritical superstrings*, *Phys. Lett.* **B 251** (1990) 67.
- [37] D. Kutasov, *Some properties of (non)critical strings*, [hep-th/9110041](#).
- [38] A. Giveon, D. Kutasov and O. Pelc, *Holography for non-critical superstrings*, *JHEP* **10** (1999) 035 [[hep-th/9907178](#)].

- [39] D. Israel and V. Niarchos, *Orientifolds in $N = 2$ Liouville theory and its mirror*, [hep-th/0703151](#).
- [40] N. Itzhaki, D. Kutasov and N. Seiberg, *Non-supersymmetric deformations of non-critical superstrings*, *JHEP* **12** (2005) 035 [[hep-th/0510087](#)].
- [41] T. Harmark, V. Niarchos and N.A. Obers, *Stable non-supersymmetric vacua in the moduli space of non-critical superstrings*, *Nucl. Phys.* **B 759** (2006) 20 [[hep-th/0605192](#)].
- [42] I. Brunner and K. Hori, *Orientifolds and mirror symmetry*, *JHEP* **11** (2004) 005 [[hep-th/0303135](#)].
- [43] S. Murthy, *Notes on non-critical superstrings in various dimensions*, *JHEP* **11** (2003) 056 [[hep-th/0305197](#)].
- [44] D. Israel, C. Kounnas, A. Pakman and J. Troost, *The partition function of the supersymmetric two-dimensional black hole and little string theory*, *JHEP* **06** (2004) 033 [[hep-th/0403237](#)].
- [45] S.K. Ashok, S. Murthy and J. Troost, *D-branes in non-critical superstrings and minimal super Yang-Mills in various dimensions*, *Nucl. Phys.* **B 749** (2006) 172 [[hep-th/0504079](#)].
- [46] S. Murthy and J. Troost, *D-branes in non-critical superstrings and duality in $N = 1$ gauge theories with flavor*, *JHEP* **10** (2006) 019 [[hep-th/0606203](#)].
- [47] S. Murthy, *On supersymmetry breaking in string theory from gauge theory in a throat*, [hep-th/0703237](#).
- [48] S.K. Ashok, S. Murthy and J. Troost, *D-branes in unoriented non-critical strings and duality in $SO(N)$ and $Sp(N)$ gauge theories*, [hep-th/0703148](#).
- [49] K. Hosomichi, *$N = 2$ Liouville theory with boundary*, *JHEP* **12** (2006) 061 [[hep-th/0408172](#)].
- [50] O. Bergman and M.R. Gaberdiel, *A non-supersymmetric open-string theory and S-duality*, *Nucl. Phys.* **B 499** (1997) 183 [[hep-th/9701137](#)].
- [51] E. Kiritsis and C. Kounnas, *Infrared regularization of superstring theory and the one loop calculation of coupling constants*, *Nucl. Phys.* **B 442** (1995) 472 [[hep-th/9501020](#)].
- [52] C. Bachas, N. Couchoud and P. Windey, *Orientifolds of the 3-sphere*, *JHEP* **12** (2001) 003 [[hep-th/0111002](#)].
- [53] S. Förste, D. Ghoshal and S. Panda, *An orientifold of the solitonic fivebrane*, *Phys. Lett.* **B 411** (1997) 46 [[hep-th/9706057](#)].
- [54] D. Burshtyn, S. Elitzur and Y. Mandelbaum, *Probing orientifold behavior near ns branes*, *JHEP* **03** (2004) 022 [[hep-th/0311188](#)].
- [55] E. Witten, *Baryons in the $1/N$ expansion*, *Nucl. Phys.* **B 160** (1979) 57.
- [56] S. Coleman, *$1/N$, in aspects of symmetry*, Cambridge University Press, Cambridge (1985).
- [57] G. 't Hooft, *A planar diagram theory for strong interactions*, *Nucl. Phys.* **B 72** (1974) 461.
- [58] T. Banks, *Matrix theory*, *Nucl. Phys.* **67** (Proc. Suppl.) (1998) 180 [[hep-th/9710231](#)].